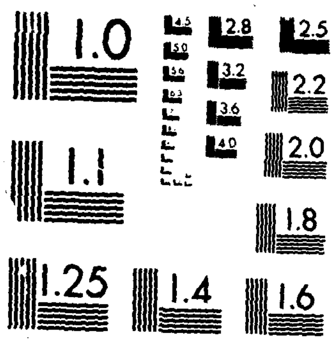
AD-A171 692 IMPROVEMENT OF RESOLUTION AND REDUCTION OF COMPUTATION 1/1
IN 2D SPECTRAL EST (U) PRINCETON UNIV NJ DEPT OF
ELECTRICAL ENGINEERING AND COMPUTER L ZOU ET AL.
UNCLASSIFIED MAR 84 AFOSR-TR-86-0642 AFOSR-81-0186 F/G 9/4 NL


1.0
4.5
5.0
5.6
6.3
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(2)

AD-A171 692

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR- 86-0642	2. GOVT ACCESSION NO. N/A
3. RECIPIENT'S CATALOG NUMBER N/A	
4. TITLE (and Subtitle) Improvement of Resolution and Reduction of Computation in 2D Spectral Estimation Using Decimation	5. TYPE OF REPORT & PERIOD COVERED Reprint
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Lihe Zou and Bede Liu	8. CONTRACT OR GRANT NUMBER(s) ~~AF~~-AFOSR-81-0186
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Electrical Engineering, Princeton University, Princeton, NJ 08544	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS ~~N/A~~ 61102F 2304 A6
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research, Bolling Air Force Base, DC 20332 nm	12. REPORT DATE March 1984
	13. NUMBER OF PAGES 4
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Same as 11	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

DTIC ELECTE SEP 15 1986 S D B

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Presented at the IEEE International Conference on Acoustics, Speech and Signal Processing, March 1984.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is concerned with spectral estimation of a finite number of two dimensional siunsoids embedded in white noise. Closed form expressions are derived for estimates using the autoregressive (AR) prediction error filter approach, as well as using periodogram with Bartlett window, and the maximum likelihood (ML) method. These expressions are useful in the study of resolving closely spaced sinusoidal signals. Over a narrow frequency band, direct decimation can be applied to improve resolution and/or to reduce com--
Continued

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-LF-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DTIC FILE COPY

86 9 15 095

Continued

putation. Simulation results demonstrate that decimation by (D_1, D_2) with a support size (N_1, N_2) yields approximately the same resolution as a support size $(D_1 N_1, D_2 N_2)$ used with the undecimated signal. The use of decimation also reduces significantly computation.

AFOSR-TR- 86-0642

Improvement of Resolution and Reduction of Computation in 2D Spectral Estimation Using Decimation*

Lihe Zou** and Bede Liu

Department of Electrical Engineering and Computer Science
Princeton University, Princeton, NJ 08544

Abstract

This paper is concerned with spectral estimation of a finite number of two dimensional sinusoids embedded in white noise. Closed form expressions are derived for estimates using the autoregressive (AR) prediction error filter approach, as well as using periodogram with Bartlett window, and the maximum likelihood (ML) method. These expressions are useful in the study of resolving closely spaced sinusoidal signals. Over a narrow frequency band, direct decimation can be applied to improve resolution and/or to reduce computation. Simulation results demonstrate that decimation by (D_1,D_2) with a support size (N_1,N_2) yields approximately the same resolution as a support size (D_1N_1,D_2N_2) used with the undecimated signal. The use of decimation also reduces significantly computation.

I. Introduction

Computation rate and the ability to resolve closely located spectral components are of concern to almost all spectral estimation methods. These problems have received considerable attention in the literature [1-4]. In this paper, we extend some of these results to the two dimensional case. Specifically, we investigate spectral estimates of a finite number of sinusoids embedded in white noise in two dimension. We shall concentrate our discussion on the use of autoregressive (AR) prediction error filter approach. Similar results can be derived for the maximum likelihood (ML) estimates and the periodogram using Bartlett window.

II. Two-Dimensional AR Spectral Estimation

The two-dimensional process under study is a sampled homogeneous (stationary) random field $\{x(n_1,n_2)\}$. Its autocorrelation function is defined as

$$r(n_1,n_2) = E\left[x(k_1+n_1,k_2+n_2)\,x^*(k_1,k_2)\right] \qquad (2.1)$$

where E denotes expectation and * indicates complex conjugate. The power spectral density is

$$P(\zeta,\xi) = \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} r(n_1,n_2)\,e^{-j(n_1\zeta+n_2\xi)}$$

$$-\pi \le \zeta,\xi \le \pi \qquad (2.2)$$

In practice, one observes $\{x(n_1,n_2)\}$ over a finite support: $1 \le n_1 \le L_1$, $1 \le n_2 \le L_2$. An estimate of the autocorrelation function can be calculated based on the observed data, and power spectral density estimate is then obtained. For simplicity, we shall use the same notation $r(n_1,n_2)$ and $P(\zeta,\xi)$ to denote these functions as well as their estimates.

Let (N_1,N_2) defines a rectangular support over which the autocorrelation function $r(n_1,n_2)$ is estimated. It is convenient to define the following notations.

$$U_\zeta = \left[1, e^{j\zeta}, e^{j2\zeta}, \cdots, e^{j(N_1-1)\zeta}\right]^T,$$

$$U_\xi = \left[1, e^{j\xi}, e^{j2\xi}, \cdots, e^{j(N_2-1)\xi}\right]^T,$$

$$U = U_\zeta \otimes U_\xi, \qquad (2.3)$$

where $\otimes$ denotes the direct product [5], and the superscript T denotes transpose. The N_1N_2 column vector U can be written as

$$U = \left[U_\xi^T, e^{j\zeta}U_\xi^T, e^{j2\zeta}U_\xi^T, \cdots, e^{j(N_1-1)\zeta}U_\xi^T\right]^T \qquad (2.4)$$

That is, its k^{th} element, $0 \le k < N_1N_2$, is $e^{j(n\zeta+l\xi)}$ where $k=nN_2+l$ with $0 \le n < N_1$, $0 \le l < N_2$. It is convenient to use the two indices (n,l) rather than the single index k. We shall say that the $(n,l)^{th}$ element of U is

$$U_{(n,l)} = e^{j(n\zeta+l\xi)},\ 0\le n<N_1,\ 0\le l<N_2 \qquad (2.5)$$

even though U is a column vector.

Let X be the column vector of size N_1N_2 whose k^{th} element is $x(n_0+n, l_0+l)$, where the index pair (n,l) is related to k as before, and (n_0,l_0) are arbitrary. We define the autocorrelation matrix R as

$$R = E\left[XX^{*T}\right] \qquad (2.6)$$

Thus the element of R at the $(n,l)^{th}$ row and the $(m,k)^{th}$ column is

$$R_{(n,l),(m,k)} = r(n-m, l-k).$$

$$0\le n,m<N_1,\ 0\le l,k<N_2. \qquad (2.7)$$

Using the autoregressive (AR) prediction error filter method [2], [6] the signal is assumed to fit an AR model of order (N_1-1, N_2-1) driven by a white noise $u(n_1,n_2)$. It can be written as

$$x(n_1,n_2) = -\sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1} a_{kl}\,x(n_1-k, n_2-l) + u(n_1,n_2) \qquad (2.8)$$

where the double summation does not include the $k=l=0$ term. The coefficients a_{kl} are estimated by minimizing the one step prediction error

$$|e(n_1,n_2)|^2 = |x(n_1,n_2) + \sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1} a_{kl}\,x(n_1-k, n_2-l)|^2 \qquad (2.9)$$

This minimization leads to the normal equation

$$RA^* = e_p\,\varepsilon \qquad (2.10)$$

where R is the autocorrelation matrix given by (2.6), A^* is a N_1N_2 column coefficient vector whose kN_2+l element is a_{kl} with $a_{00}=1$, ε is the N_1N_2 column vector

$$\varepsilon = [1, 0 \cdots 0]^T \qquad (2.11)$$

and e_p is the prediction error power, a scalar. The spectrum is given by

$$P_{AR}(\zeta,\xi) = \frac{e_p}{\left|\sum_{k=0}^{N_1-1}\sum_{l=0}^{N_2-1} a_{kl}\,e^{-j(k\zeta+l\xi)}\right|^2} \qquad (2.12)$$

* This work was supported by the Air Force Office of Scientific Research under grant AF-AFOSR 81-0186

** On leave from Jiao Tong University, Xian, People's Republic of China

and can be expressed as

$$P_{AR}(\zeta,\xi) = \frac{\varepsilon^T R^{-1}\varepsilon}{|U^T R^{-1}\varepsilon|^2} \tag{2.13}$$

where U is given by Eq. (2.3).

It is worth noting that the 2-D autocorrelation matrix R defined above is a symmetric, positive definite and ***blockToeplitz***, but not Toeplitz.

III. Sinusoidal Signals in White Noise

In order to study the resolution characteristics of an AR spectral estimation, we assume that true values of the autocorrelation function are known rather than those obtained from actual data. The signal under study is composed of a finite number, K, sinusoids and a white noise with unit power. The autocorrelation function is

$$r(n_1,n_2) = \delta(n_1,n_2) + \sum_{k=1}^{K} \rho_k e^{j(n_1\zeta_k + n_2\xi_k)} \tag{3.1}$$

where (ζ_k,ξ_k) is the frequency of the k^{th} sinusoid and ρ_k the corresponding power. The matrix R on a support (N_1,N_2) has the form

$$R = I + \sum_{k=1}^{K} \rho_k U_k^* U_k^T \tag{3.2}$$

where I is the (N_1N_2) square identity matrix and U_k is a N_1N_2 column vector identical to U of Eq. (2.3) but with (ζ_k,ξ_k) in place of (ζ,ξ).

It can be shown that the AR spectrum in this case is given by

$$P_{AR}(\zeta,\xi) = \frac{1-\sum_{l=1}^{K} d_l}{|1-N_1N_2\sum_{l=1}^{K} d_l\,\varphi_{N_1N_2}(\zeta-\zeta_l,\xi-\xi_l)|^2} \tag{3.3}$$

where d_l are constants and

$$\varphi_{N_1N_2}(\zeta,\xi) = B_{N_1}(\zeta)B_{N_2}(\xi) \tag{3.4}$$

with $B_N(\lambda)$ given by

$$B_N(\lambda) = \frac{1}{N}\sum_{n=0}^{N-1} e^{jn\lambda} = e^{j(\frac{N-1}{2})\lambda}\frac{\sin(N\lambda/2)}{N\sin(\lambda/2)}. \tag{3.5}$$

In the case of $K=1$,

$$R = I + \rho U_1^* U_1^T, \tag{3.6}$$

and Eq. (3.3) reduces to

$$P_{AR}(\zeta,\xi) = \frac{1-\rho/(1+N_1N_2\rho)}{|1-N_1N_2\rho\varphi_{N_1N_2}(\zeta-\zeta_1,\xi-\xi_1)/(1+N_1N_2\rho)|^2}. \tag{3.7}$$

It has a peak at the unbiased location (ζ_1,ξ_1), and the peak value is

$$P_{AR}(\zeta_1,\xi_1) = (1+N_1N_2\rho)[1+(N_1N_2-1)\rho] \doteq (N_1N_2\rho)^2 \tag{3.8}$$

which is proportional to ρ^2. So the peak of AR spectrum is not a power estimate but a square power estimate.

We now determine the 3 dB contour around the peak in the frequency plane (ζ,ξ) from the equation

$$P_{AR}(\zeta_1,\xi_1) = 2\,P_{AR}(\zeta,\xi) \tag{3.9}$$

By using first 2 terms of the Taylor series expansion of Eq. (3.3), it can be shown that the contour is approximately given by

$$|(N_1-1)(\zeta-\zeta_1) + (N_2-1)(\xi-\xi_1)| = 2/N_1N_2\rho \tag{3.10}$$

which is a rhombus with the "major/minor axes" equal to

$$D_{\zeta,AR} = 4/(N_1-1)N_1N_2\rho,$$

and

$$D_{\xi,AR} = 4/(N_2-1)N_1N_2\rho. \tag{3.11}$$

This is plotted in Fig. 1, along with some simulation results. The data size is $L_1 = L_2 = 64$, and the relevant parameters are: $N_1 = N_2 = 5$ and $\zeta_1 = \xi_1 = 0.5\pi$.

For two sinusoids ($K=2$) in white noise, Eq. (3.3) becomes

$$P_{AR}(\zeta,\xi) = \frac{1-d_1-d_2}{|1-N_1N_2d_1\varphi_{N_1N_2}(\alpha_1,\beta_1) - N_1N_2d_2\varphi_{N_1N_2}(\alpha_2,\beta_2)|^2} \tag{3.12}$$

where $\alpha_1=\zeta-\zeta_1$, $\alpha_2=\zeta-\zeta_2$ and $\beta_1=\xi-\xi_1$, $\beta_2=\xi-\xi_2$ and $\{d_i\}$ depend on the signal powers $\{\rho_i\}$ as well as the frequency separations $(\zeta_1-\zeta_2)$ and $(\xi_1-\xi_2)$. These expressions show that the P_{AR} are not linear with respect to the individual components, and that there is always interference between them. The effect of this interference on resolution is not obvious. Roughly speaking, however, when the two frequency components are close to each other with respect to the 3 dB axes, then the two spectral peaks will merge. Since the 3 dB axes are decreasing function of the signal power as well as the size of support, increasing signal power and/or increasing the size of support will improve resolution.

We note in passing that if $N_2=1$ and $\xi=0$, the above analysis reduces to a one-dimensional case [1], [4].

IV. Other Spectral Estimates

The above discussion can be modified in a straightforward way to be applicable to the periodogram using Bartlett window P_B [7] and the maximum likelihood estimates P_{ML} [8]. It can be shown that these estimates are given respectively by

$$P_B(\zeta,\xi) = \frac{1}{N_1^2N_2^2}U^T R U^*, \tag{4.1}$$

and

$$P_{ML}(\zeta,\xi) = \frac{1}{U^T R^{-1} U^*}. \tag{4.2}$$

For K sinusoids in white noise, these expressions reduce to

$$P_B(\zeta,\xi) = \frac{1}{N_1N_2} + \sum_{k=1}^{K} \rho_k|\varphi_{N_1N_2}(\alpha_k,\beta_k)|^2, \tag{4.3}$$

and

$$P_{ML}(\zeta,\xi) = \frac{1/N_1N_2}{1-N_1N_2\sum_{l=1}^{K}\sum_{m=1}^{K} C_{lm}\varphi_{N_1N_2}(\alpha_l,\beta_l)\varphi^*_{N_1N_2}(\alpha_m,\beta_m)} \tag{4.4}$$

The coefficients C_{lm} depend on the signal powers and the frequency separations.

V. Decimation to Improve Resolution

It is seen in the previous section that the resolution can be improved by increasing the size of support. In the 2-D case, however, the increasing size of support will greatly increase the computation since the size of autocorrelation matrix R is $(N_1N_2)\times(N_1N_2)$. We demonstrate in this section that direct decimation of an input data sequence can improve the resolution without increasing the size of support. This technique has been used in the one-dimensional case on the periodogram [3], the ML (Capon) method, and the AR method [4]. It is to be expected that the saving in computation in the 2-D case is even more significant.

The direct decimation scheme is depicted in Fig. 2. The two reasons for the higher resolution are that the decimation expands the frequency scales by the factor D_1 and D_2 respectively and that the bandpass filter removes interference from out-of-band noise, thus raising the

effective SNR.

To demonstrate the improvement of resolution by decimation, Fig. 3 shows the AR estimates on the data which consists of 2 sinusoids at frequencies $(0.1775\pi, 0.1775\pi)$ and $(0.1975\pi, 0.1975\pi)$ and noise 7 dB below the sinusoids. Fig. 5(A) shows the P_{AR} with $N_1 = N_2 = 5$ without decimation, (B) the $P_{AR,D}$ with $N_1 = N_2 = 5$ and decimation $D_1 = D_2 = 4$, both plotted on $0.125\pi \le \zeta, \xi \le 0.25\pi$. These simulation results show that direct decimation improves the resolution without increasing the size of support.

In order to investigate the improvement of resolution by decimation, we may introduce the notation of "resolution boundary" which, as in the 1-D case [9], for two sinusoids with equal power is defined as the frequency separation $(\Delta\zeta, \Delta\xi) = (|\zeta_1-\zeta_2|, |\xi_1-\xi_2|)$ at which the spectrum at the center frequency is equal to the average of the spectra at the two sinusoid frequencies, i.e.,

$$P_{AR}\left(\frac{\zeta_1+\zeta_2}{2}, \frac{\xi_1+\xi_2}{2}\right) = \frac{1}{2}[P_{AR}(\zeta_1, \xi_1) + P_{AR}(\zeta_2, \xi_2)] \quad (5.1)$$

The resolution boundary is the minimum resolvable frequency separation for a given SNR. In Fig. 4 we show a special symmetric case with $\Delta\zeta = \Delta\xi$. The solid line exhibits the theoretical curve of SNR vs. $\Delta\zeta$ computed from (5.1) with $N_1 = N_2 = 10$. The dashed line indicates the same theoretical curve for corresponding decimated spectra with $N_1 = N_2 = 5$, and $D_1 = D_2 = 2$. The two curves are seen to be close to each other. The triangles and rectangles are corresponding simulation results on a (32×32) data with center frequency $\zeta_0 = \xi_0 = 0.25\pi$. Since these results are obtained by averaging only two independent trials, a considerable variation is present.

VI. Decimation to Reduce Computation

In the last section, we have seen that using decimation by factor (D_1, D_2) can reduce the support size (N_1, N_2) to $(\frac{N_1}{D_1}, \frac{N_2}{D_2})$ while maintaining the same resolution. A reduction in the size of support is accompanied by a saving of computation.

For a support (N_1, N_2) and a data size $(L_1 \times L_2)$, the number of multiplication in the computation of autocorrelation matrix is

$$M_R = N_1N_2(L_1-\frac{N_1+1}{2})(L_2-\frac{N_2+1}{2}) \approx N_1N_2L_1L_2 \quad (6.1)$$

If the Justic algorithm [10] is used to solve the block Toeplitz matrix normal equation, Eq. (2.10), the computation may be $0(N_1^3N_2^3)$. If the Gaussian-Seidel iteration is used, the number of multiplication is

$$M_N \approx \gamma(N_1N_2)^2 \quad (6.2)$$

where γ is a constant depended on the number of iteration. Thus the total number of multiplication for a P_{AR} estimate is

$$M_{AR} = M_R + M_N \approx N_1N_2L_1L_2 + \gamma N_1^2N_2^2 \quad (6.3)$$

Suppose the band-pass filter preceding the down sampling in Fig. 2 is a first quadrant FIR filter with length of impulse response (N_{F1}, N_{F2}). The number of multiplication for the filtering is

$$M_{F,D} = \frac{(L_1+N_{F1})N_{F1}}{2D_1}\frac{(L_2+N_{F2})N_{F2}}{2D_2} \quad (6.4)$$

The number of multiplication for computing the autocorrelation matrix and for solving normal equation for a decimated AR spectrum are respectively

$$M_{R,D} = \frac{N_1N_2}{D_1D_2}\left[\frac{L_1+N_{F1}}{D_1} - \frac{N_1+D_1}{2D_1}\right]\left[\frac{L_2+N_{F2}}{D_2} - \frac{N_2+D_2}{2D_2}\right] \approx \frac{M_R}{(D_1D_2)^2} \quad (6.5)$$

and $$M_{N,D} = \gamma\frac{(N_1N_2)^2}{(D_1D_2)^2} = \frac{M_N}{(D_1D_2)^2} \quad (6.6)$$

Thus, the total number of multiplication is approximately

$$M_{AR,D} = M_{R,D} + M_{N,D} + M_{F,D} = \frac{M_{AR}}{(D_1D_2)^2} + M_{F,D} \quad (6.7)$$

If $D_1 = D_2 = D$, the saving of computation is by a factor D^4, excluding the filtering $M_{F,D}$

Discussion

The resolution of sinusoids in white noise using 2D AR spectrum is investigated in this paper. Closed form expressions of spectral estimates are given. The peaks of the AR spectra indicate the square of the power of each component. A 3 dB contour in the frequency plane is introduced to facilitate the study of resolution characteristics.

Decimation is then applied to narrow-band 2-D spectral estimation. Simulation results indicate that a spectral estimate on a support (N_1N_2) with a decimation factor of (D_1, D_2) has a resolution approximately equivalent to that of a spectral estimate on a support (D_1N_1, D_2N_2) without decimation. It is also shown that decimation reduces computation by a factor $(D_1D_2)^2$ without sacrificing resolution.

This analysis can be extended to most other 2-D spectral estimation methods, such as periodogram and maximum likelihood method, and similar results can be obtained.

References

1. R.T. Laccoss, "Data Adaptive Spectral Analysis Method," *Geophys.*, Vol. 36, pp. 661-675, Aug. 1971.
2. A.K. Jain and S. Raganath, "High Resolution Spectrum Estimation in Two Dimensions," *in Proc. 1st ASSP Workshop on Spectral Estimation*, (Hamilton, Ont., Canada, Aug. 1981), pp. 3.4.1-3.4.5.
3. B. Liu and F. Mintzer, "Calculation of Narrow-Band Spectra by Direct Decimation," *IEEE Trans. Acous. Sp. and Sig. Proc.*, Vol. ASSP-26, No. 6, pp. 529-534, Dec. 1978.
4. M. Quirk and B. Liu, "Improving Resolution for Autoregressive Spectral Estimation by Decimation," *IEEE Trans. Acous. Sp. and Sig. Proc.*, Vol. ASSP-31, pp. 630-637, June 1983.
5. P. Lancaster, Theory of Matrices, Academic Press, New York and London, 1969.
6. J.H. McClellan, "Multidimensional Spectral Estimation," *Proc. of the IEEE*, Vol. 70, No. 9, Sept. 1982.
7. P.D. Welch, "The Use of FFT for the Estimation of Power Spectra: A Method Based on Time Average over Short, Modified Periodograms," *IEEE Trans. Audio Electroacoust.*, Vol. AU-15, pp. 70-73, June 1967.
8. J. Capon, "High-Resolution Frequency-Wavenumber Spectrum Analysis," *Proc. IEEE*, Vol. 57, pp. 1408-1418, Aug. 1969.
9. L. Marple, "Resolution of Conventional Fourier, Autoregressive and Special ARMA Method of Spectrum Analysis," *Proc. IEEE Int. Conf. Acoust. Sp. and Sig. Proc.*, pp. 74-77, (Hartford, CT, 1977).
10. J.H. Justice,"A Levison-Type Algorithm for Two-Dimensional Weiner Filtering Using Bivariate Szego Polynomials," *Proc. IEEE*, Vol. 65, No. 6, pp. 882-886, June 1977.

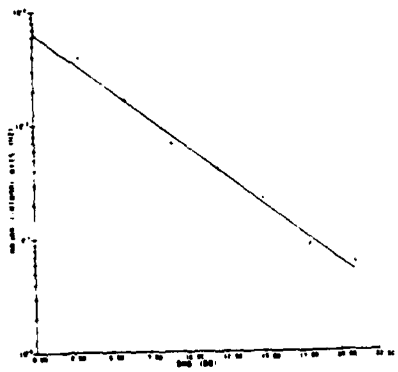


FIG 1 3DB MAJOR/MINOR AXES FOR 2-D AR SPECTRUM

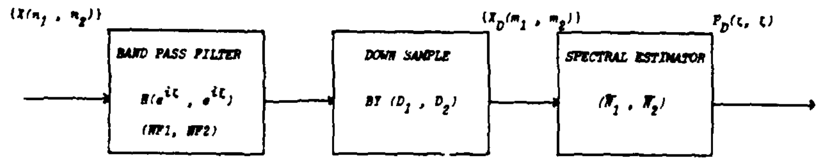


FIGURE 2 - Direct Decimation Spectral Estimation

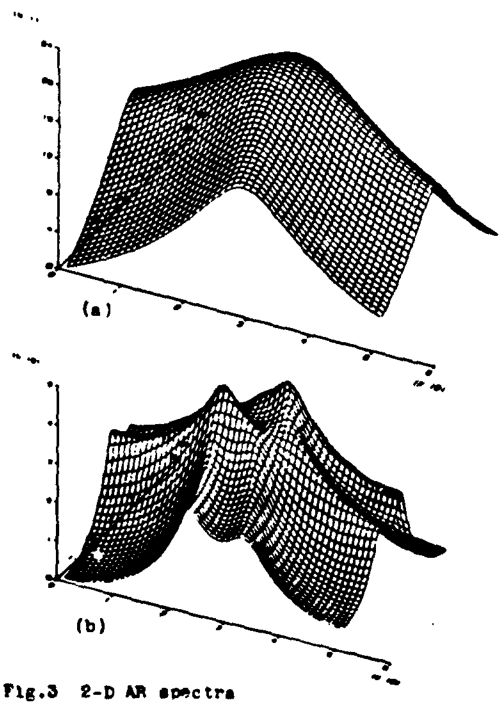


Fig.3 2-D AR spectra
(a) without decimation
(b) with decimation

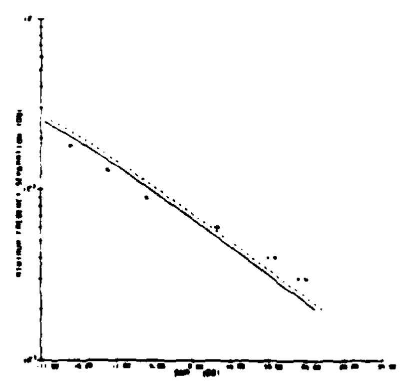


FIG4 RESOLUTION BOUNDARY FOR 2-D AR SPECTRUM

END
DTIC
11-86